CHAPTER

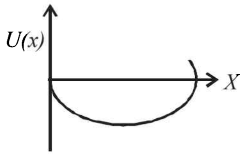
4

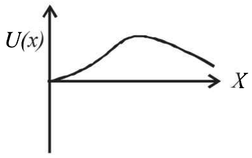
Work, Energy and Power

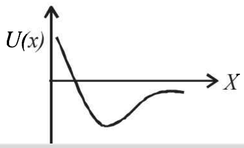
Section-A JEE Advanced/ IIT-JEE

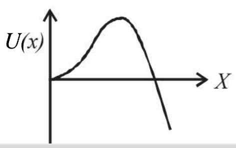
C MCQs with One Correct Answer

1. If a machine is lubricated with oil *(1980)*
   (a) the mechanical advantage of the machine increases.
   (b) the mechanical efficiency of the machine increases.
   (c) both its mechanical advantage and efficiency increase.
   (d) its efficiency increases, but its mechanical advantage decreases.
2. Two masses of 1 gm and 4 gm are moving with equal kinetic energies. The ratio of the magnitudes of their linear momenta is *(1980)*
   (a) 4 : 1 (b) $\sqrt{2}:1$ (c) 1 : 2 (d) 1 : 16
3. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$ where k is a constant. The power delivered to the particles by the force acting on it is: *(1994 - 1 Mark)*
   (a) $2\pi\, mk^2r^2\, t$ (b) mk^2r^2t
   (c) $\frac{(mk^4r^2t^5)}{3}$ (d) zero
4. A spring of force-constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force-constant of *(1999S - 2 Marks)*
   (a) $(2/3)k$ (b) $(3/2)k$ (c) $3\,k$ (d) $6\,k$
5. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to *(2000S)*
   (a) v (b) v^2 (c) v^3 (d) v^4
6. A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is *(2002S)*

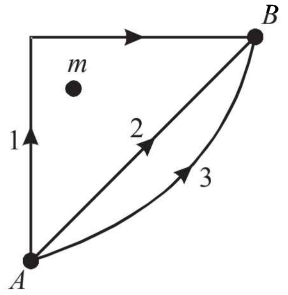
(a) 

(b) 

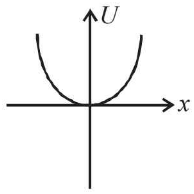
(c) 

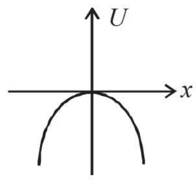
(d) 

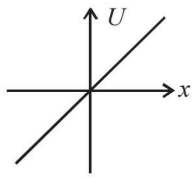
7. An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is *(2002S)*
   (a) $\frac{4Mg}{k}$ (b) $\frac{2Mg}{k}$ (c) $\frac{Mg}{k}$ (d) $\frac{Mg}{2k}$
8. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1,2 and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3 *(2003S)*
   (a) $W_1 > W_2 > W_3$
   (b) $W_1 = W_2 = W_3$
   (c) $W_1 < W_2 < W_3$
   (d) $W_2 > W_1 > W_3$

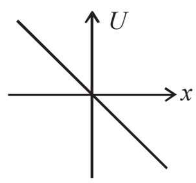


9. A particle is acted by a force $F = kx$, where k is a +ve constant. Its potential energy at $x = 0$ is zero. Which curve correctly represents the variation of potential energy of the block with respect to x *(2004S)*

(a) 

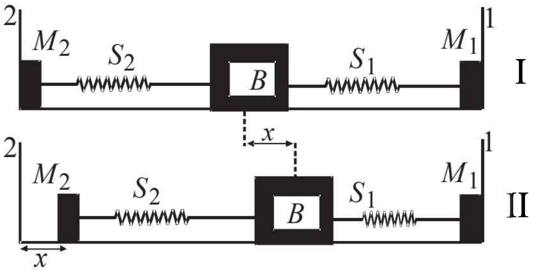
(b) 

(c) 

(d) 

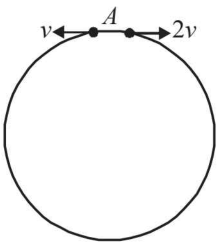
10. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$, respectively (see fig. I). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports

have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio y/x is – ***(2008)***



(a) 4 (b) 2 (c) 1/2 (d) 1/4

11. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A? ***(2009)***

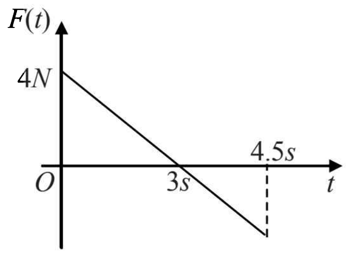


(a) 4 (b) 3 (c) 2 (d) 1

12. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is ***(2009)***

(a) $\frac{a}{gk}$ (b) $\frac{a}{2gk}$ (c) $\frac{2a}{gk}$ (d) $\frac{a}{4gk}$

13. A block of mass 2 kg is free to move along the x-axis. It is at rest and from t = 0 onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is ***(2010)***



(a) 4.50 J

(b) 7.50 J

(c) 5.06 J

(d) 14.06 J

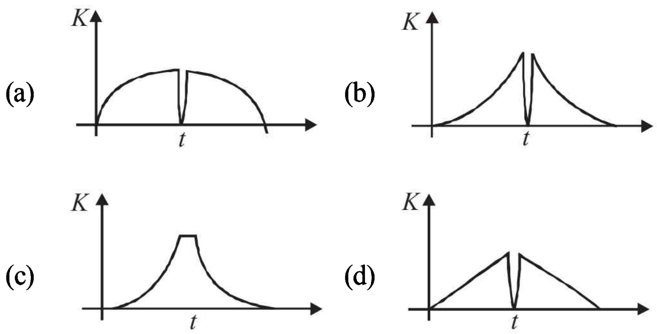
14. The work done on a particle of mass m by a force,

$$K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i}+\frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$$

(K being a constant of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the $x-y$ plane is ***(JEE Adv. 2013)***

(a) $\frac{2K\pi}{a}$ (b) $\frac{K\pi}{a}$ (c) $\frac{K\pi}{2a}$ (d) 0

15. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figure are only illustrative and not to the scale. ***(JEE Adv. 2014)***



D MCQs with One or More than One Correct

1. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to ***(1984- 2 Marks)***

(a) $t^{1/2}$ (b) $t^{3/4}$ (c) $t^{3/2}$ (d) t^2

2. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is ***(1985 - 2 Marks)***

(a) MgL (b) $MgL/3$ (c) $MgL/9$ (d) $MgL/18$

3. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that : ***(1987 - 2 Marks)***

(a) its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant.
(d) it moves in a circular path.

4. A force $F = -K(y\hat{i} + x\hat{j})$ (where K is a positive constant) acts on a particle moving in the xy plane. Starting from the origin, the particle is taken along the positive x axis to the point $(a, 0)$, and then parallel to the y axis to the point (a, a), The total work done by the force F on the particle is ***(1998S - 2 Marks)***

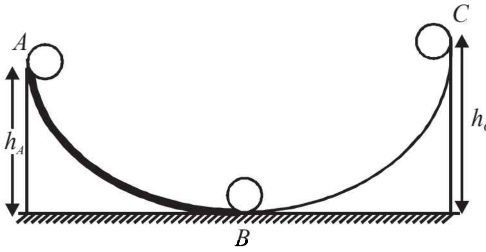
(a) $-2Ka^2$ (b) $2Ka^2$ (c) $-Ka^2$ (d) Ka^2

5. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position, and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is *(1998S - 2 Marks)*

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$

(c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$

6. A small ball starts moving from A over a fixed track as shown in the figure. Surface AB has friction. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C, respectively. Then *(2006 - 5M, –1)*



(a) $h_A > h_C;\ K_B > K_C$ (b) $h_A > h_C;\ K_C > K_A$

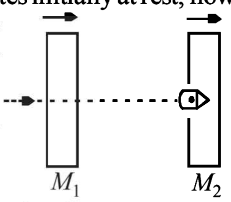
(c) $h_A = h_C;\ K_B = K_C$ (d) $h_A < h_C;\ K_B > K_C$

E Subjective Problems

1. A bullet is fired from a rifle. If the rifle recoils freely, determine whether the kinetic energy of the rifle is greater than, equal or less than that of the bullet. *(1978)*

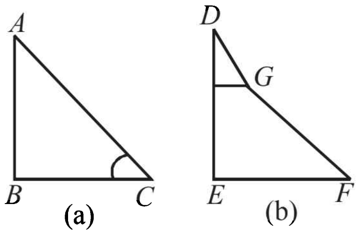
2. A spring of force constant k is cut into three equal parts. What is force constant of each part? *(1978)*

3. A 20 gm bullet pierces through a plate of mass $M_1 = 1$ kg and then comes to rest inside a second plate of mass $M_2 = 2.98$ kg. as shown. It is found that the two plates initially at rest, now move with equal velocities. Find the percentage loss in the initial velocity of the bullet when it is between M_1 and M_2. Neglect any loss of material of the plates due to the action of the bullet. *(1979)*



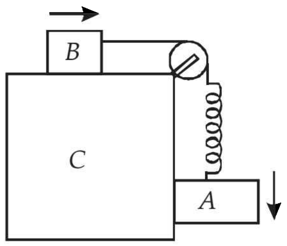
4. When a ball is thrown up, the magnitude of its momentum decreases and then increases. Does this violate the conservation of momentum principle? *(1979)*

5.



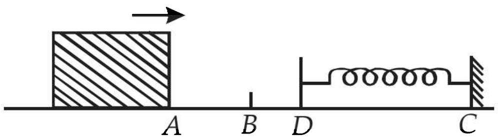
In the figures (a) and (b) AC, DG and GF are fixed inclined planes, $BC = EF = x$ and $AB = DE = y$. A small block of mass M is released from the point A. It slides down AC and reaches C with a speed V_c. The same block is released from rest from the point D. It slides down DGF and reaches the point F with speed V_F. The coefficients of kinetic frictions between the block and both the surface AC and DGF are μ. *(1980)* Calculate V_C and V_F.

6. Two blocks A and B are connected to each other by a string and a spring ; the string passes over a frictionless pulley as shown in the figure. Block B slides over the horizontal top surface of a stationary block C and the block A slides along the vertical side of C, both with the same uniform speed.

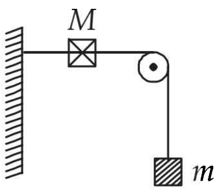


The coefficient of friction between the surfaces of blocks is 0.2. Force constant of the spring is 1960 newtons/m. If mass of block A is 2 Kg., calculate the mass of block B and the energy stored in the spring. *(1982 - 5 Marks)*

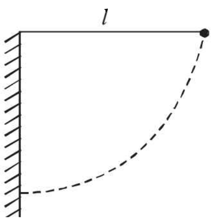
7. A 0.5 kg block slides from the point A (see Fig) on a horizontal track with an initial speed of 3 m/s towards a weightless horizontal spring of length 1 m and force constant 2 Newton/m. The part AB of the track is frictionless and the part BC has the coefficients of static and kinetic friction as 0.22 and 0 2 respectively. If the distances AB and BD are 2 m and 2.14 m respectively, find the total distance through which the block moves before it comes to rest completely. (Take $g = 10 m/s^2$) *(1983 - 7 Marks)*



8. A string, with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2m from the wall, has a point mass $M = 2$kg attached to it at a distance of 1m from the wall. A mass $m = 0.5$ kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley and vertical beyond the pulley. What will be the speed with which the mass M will hit the wall when the mass m is released? *(1985 - 6 Marks)*



9. A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position (see fig.) and released. The ball hits the wall, the coefficient of restitution being $\frac{2}{\sqrt{5}}$.



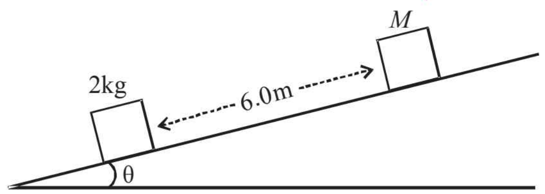
(1987 - 7 Marks)

What is the minimum number of collisions after which the amplitude of oscillations becomes less than 60 degrees ?

10. Two blocks of mass 2 kg and M are at rest on an inclined plane and are separated by a distance of 6.0 m as shown in Figure. The coefficient of friction between each of the blocks and the inclined plane is 0.25. The 2 kg block is given a velocity of 10.0 m/s up the inclined plane. It collides with M, comes back and has a velocity of 1.0 m/s when it reaches its initial position. The other block M after the collision moves 0.5 m up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block M.

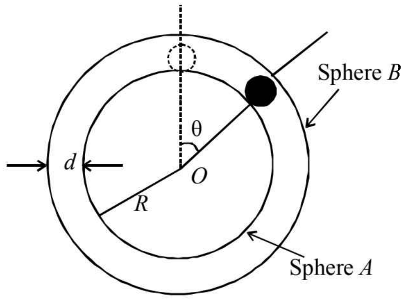
[Take $\sin\theta \approx \tan\theta = 0.05$ and $g = 10\text{m/s}^2$.]

(1999 - 10 Marks)



11. A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than d. All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by θ (shown in the figure).

(2002 - 5 Marks)



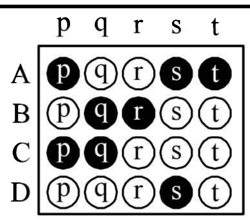
(a) Express the total normal reaction force exerted by the sphere on the ball as a function of angle θ.

(b) Let N_A and N_B denote the magnitudes of the normal reaction forces on the ball exerted by the sphere A and B, respectively. **Sketch** the variations of N_A and N_B as functions of $\cos\theta$ in the range $0 \le \theta \le \pi$ by drawing two **separate** graphs in your answer book, taking $\cos\theta$ on the horizontal axes.

F Match the Following

DIRECTIONS (Q. No. 1) : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



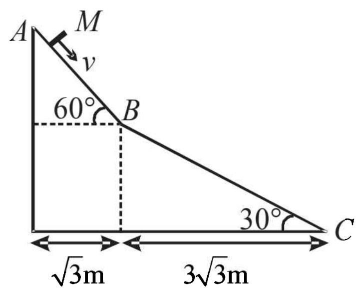
1. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 constants). Match the potential energies in column I to the corresponding statement(s) in column II.

Column I	Column II
(A) $U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(p) The force acting on the particle is zero at $x = a$
(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(q) The force acting on the particle is zero at $x = 0$
(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(r) The force acting on the particle is zero at $x = -a$
(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(s) The particle experiences an attractive force towards $x = 0$ in the region $\lvert x\rvert < a$
	(t) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = -a$

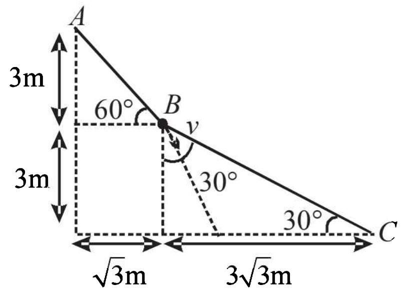
G Comprehension Based Questions

PASSAGE-1

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point *B*. The block is initially at rest at *A*. Assume that collisions between the block and the incline are totally inelastic ($g = 10\ m/s^2$). ***(2008)***



1. The speed of the block at point *B* immediately after it strikes the second incline is –



(a) $\sqrt{60}$ m/s (b) $\sqrt{45}$ m/s

(c) $\sqrt{30}$ m/s (d) $\sqrt{15}$ m/s

2. The speed of the block at point *C*, immediately before it leaves the second incline is

(a) $\sqrt{120}$ m/s (b) $\sqrt{105}$ m/s

(c) $\sqrt{90}$ m/s (d) $\sqrt{75}$ m/s

3. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point *B*, immediately after it strikes the second incline is –

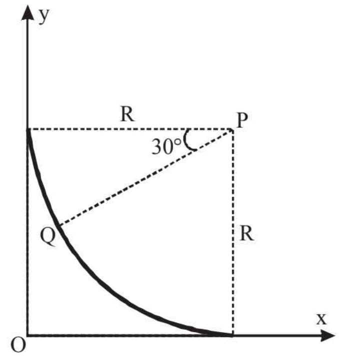
(a) $\sqrt{30}$ m/s (b) $\sqrt{15}$ m/s

(c) 0 (d) $-\sqrt{15}$ m/s

PASSAGE-2

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure below, is 150 J.
(Take the acceleration due to gravity, $g = 10\ ms^{-2}$)

(JEE Adv. 2013)



4. The magnitude of the normal reaction that acts on the block at the point Q is

(a) 7.5 N (b) 8.6 N

(c) 11.5 N (d) 22.5 N

5. The speed of the block when it reaches the point Q is

(a) $5\ ms^{-1}$ (b) $10\ ms^{-1}$

(c) $10\sqrt{3}\ ms^{-1}$ (d) $20\ ms^{-1}$

H Assertion & Reason Type Questions

1. **STATEMENT–1 :** A block of mass *m* starts moving on a rough horizontal surface with a velocity *v*. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity *v*. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

STATEMENT–2 : The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination. ***(2007)***

(a) Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation for Statement–1

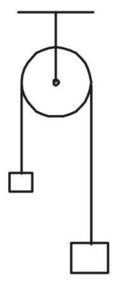
(b) Statement–1 is True, Statement–2 is True; Statement–2 is NOT a correct explanation for Statement–1

(c) Statement–1 is True, Statement–2 is False

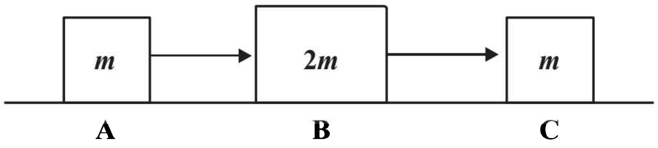
(d) Statement–1 is False, Statement–2 is True

I Integer Value Correct Type

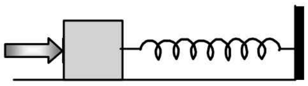
1. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10\ m/s^2$, find the work done **(in joules)** by the string on the block of mass 0.36 kg during the first second after the system is released from rest. ***(2009)***



2. Three objects *A, B* and *C* are kept in a straight line on a frictionless horizontal surface. These have masses *m, 2m* and *m,* respectively. The object *A* moves towards *B* with a speed 9 m/s and makes an elastic collision with it. There after, *B* makes completely inelastic collision with *C*. All motions occur on the same straight line. Find the final speed **(in m/s)** of the object C. *(2009)*

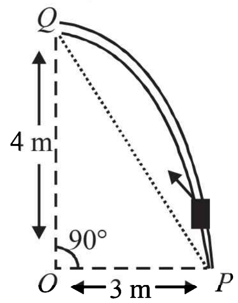


3. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V = N/10. Then N is *(2011)*



4. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is *(JEE Adv. 2013)*

5. Consider an elliptical shaped rail *PQ* in the vertical plane with *OP* = 3 m and *OQ* = 4 m. A block of mass 1 kg is pulled along the rail from *P* to *Q* with a force of 18 N, which is always parallel to line *PQ* (see the figure given). Assuming no frictionless losses, the kinetic energy of the block when it reaches *Q* is ($n \times 10$) joules. The value of *n* is (take acceleration due to gravity = 10 ms^{-2}) *(JEE Adv. 2014)*



Section-B JEE Main / AIEEE

1. Consider the following two statements : **[2003]**
   A. Linear momentum of a system of particles is zero
   B. Kinetic energy of a system of particles is zero.
   Then
   (a) A does not imply *B* and *B* does not imply *A*
   (b) *A* implies *B* but *B* does not imply *A*
   (c) *A* does not imply *B* but *B* implies *A*
   (d) *A* implies *B* and *B* implies *A*

2. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is **[2003]**
   (a) 0.2 J (b) 10 J
   (c) 20 J (d) 0.1 J

3. A spring of spring constant 5×10^3 N/m is stretched initially by 5cm from the unstretched position. Then the work required to stretch it further by another 5 cm is **[2003]**
   (a) 12.50 N-m (b) 18.75 N-m
   (c) 25.00 N-m (d) 6.25 N-m

4. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time '*t*' is proportional to **[2003]**
   (a) $t^{3/4}$ (b) $t^{3/2}$
   (c) $t^{1/4}$ (d) $t^{1/2}$

5. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement *x* is proportional to **[2004]**
   (a) x (b) e^x
   (c) x^2 (d) $\log_e x$

6. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table ? **[2004]**
   (a) 12 J (b) 3.6 J
   (c) 7.2 J (d) 1200 J

7. A force $\vec{F} = (5\vec{i} + 3\vec{j} + 2\vec{k})N$ is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\vec{i} - \vec{j})m.$ The work done on the particle in joules is **[2004]**
   (a) +10 (b) +7
   (c) –7 (d) +13

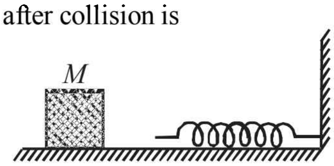
8. A body of mass 'm', accelerates uniformly from rest to 'v_1' in time 't_1'. The instantaneous power delivered to the body as a function of time 't' is [2004]

(a) $\frac{mv_1 t^2}{t_1}$ (b) $\frac{mv_1^2 t}{t_1^2}$ (c) $\frac{mv_1 t}{t_1}$ (d) $\frac{mv_1^2 t}{t_1}$

9. A Particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle, the motion of the particles takes place in a plane. It follows that [2004]

(a) its kinetic energy is constant
(b) its acceleration is constant
(c) its velocity is constant
(d) it moves in a straight line

10. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is [2005]



(a) $\frac{kL^2}{2M}$ (b) $\sqrt{Mk}\ L$ (c) $\frac{ML^2}{k}$ (d) zero

11. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is [2005]

(a) 20 m/s (b) 40 m/s
(c) $10\sqrt{30}$ m/s (d) 10 m/s

12. A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time is given by [2005]

(a) $\frac{mv^2}{T^2}.t^2$ (b) $\frac{mv^2}{T^2}.t$ (c) $\frac{1}{2}\frac{mv^2}{T^2}.t^2$ (d) $\frac{1}{2}\frac{mv^2}{T^2}.t$

13. A particle of mass 100g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is [2006]

(a) –0.5 J (b) –1.25 J
(c) 1.25 J (d) 0.5 J

14. The potential energy of a 1 kg particle free to move along the x-axis is given by $V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right) J$.

The total mechanical energy of the particle is 2 J. Then, the maximum speed (in m/s) is [2006]

(a) $\frac{3}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 2

15. A 2 kg block slides on a horizontal floor with a speed of 4m/s. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15N and spring constant is 10,000 N/m. The spring compresses by

(a) 8.5 cm (b) 5.5 cm [2007]
(c) 2.5 cm (d) 11.0 cm

16. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range [2008]

(a) 200 J - 500 J (b) 2×10^5 J - 3×10^5 J
(c) 20, 000 J - 50,000 J (d) 2,000 J - 5, 000 J

17. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is [2008]

(a) 0.16 J (b) 1.00 J
(c) 0.67 J (d) 0.34 J

18. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = \left[U(x=\infty) - U_{\text{at equilibrium}}\right]$, D is [2010]

(a) $\frac{b^2}{2a}$ (b) $\frac{b^2}{12a}$ (c) $\frac{b^2}{4a}$ (d) $\frac{b^2}{6a}$

19. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

STATEMENT 1 : If stretched by the same amount work done on S_1, Work done on S_1 is more than S_2

STATEMENT 2 : $k_1 < k_2$ [2012]

(a) Statement 1 is false, Statement 2 is true.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for Statement 1

20. When a rubber-band is stretched by a distance x, it exerts restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is: **[JEE Main 2014]**

(a) $aL^2 + bL^3$
(b) $\frac{1}{2}(aL^2 + bL^3)$
(c) $\frac{aL^2}{2} + \frac{bL^3}{3}$
(d) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

21. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$: **[JEE Main 2016]**

(a) 9.89×10^{-3} kg
(b) 12.89×10^{-3} kg
(c) 2.45×10^{-3} kg
(d) 6.45×10^{-3} kg

4 Work, Energy and Power

Section-A : JEE Advanced/ IIT-JEE

C	1.	(b)	2.	(c)	3.	(b)	4.	(b)	5.	(c)	6.	(d)
	7.	(b)	8.	(b)	9.	(b)	10.	(c)	11.	(c)	12.	(b)
	13.	(c)	14.	(d)	15.	(b)						
D	1.	(c)	2.	(d)	3.	(c, d)	4.	(c)	5.	(d)	6.	(a, b)
E	1.	Less than	2.	3 times	3.	25%						

4. No 5. $v_C = \sqrt{2g(y-\mu x)}$; $v_F = \sqrt{2g(y-\mu x)}$

6. 10 kg, 0.098 J 7. 4.24 m 8. 3.29 m/s 9. 4

10. 0.84; 15.02 kg

11. (a) $N_A = mg(3\cos\theta - 2)$

(b) For $\theta \le \cos^{-1}\left(\frac{2}{3}\right)$; $N_B = 0, N_A = mg(3\cos\theta - 2)$

For $\theta > \cos^{-1}\left(\frac{2}{3}\right)$; $N_A = 0, N_B = mg(2 - 3\cos\theta)$

F 1. (A) p, q, r, t; (B) q, s; (C) p, q, r, s; (D) p, r, t

G	1.	(b)	2.	(b)	3.	(c)	4.	(a)	5.	(b)
H	1.	(c)								
I	1.	8J	2.	4 m/s	3.	4	4.	5	5.	5

Section-B : JEE Main/ AIEEE

1.	(c)	2.	(d)	3.	(b)	4.	(b)	5.	(c)	6.	(b)	7.	(b)	8.	(b)
9.	(a)	10.	(b)	11.	(b)	12.	(b)	13.	(b)	14.	(a)	15.	(b)	16.	(d)
17.	(c)	18.	(c)	19.	(a)	20.	(c)	21.	(b)						

Section-A JEE Advanced/ IIT-JEE

C. MCQs with ONE Correct Answer

1. **(b)** Mechanical efficiency $= \frac{\text{Output work}}{\text{Input energy}}$

The output work will increase because the friction becomes less. Thus the mechanical efficiency increases.

2. **(c)** K.E. $= \frac{p^2}{2m}$

$E_1 = E_2 \quad \therefore \frac{p_1^2}{m_1} = \frac{p_2^2}{m_2}$

$\therefore \frac{p_1^2}{p_2^2} = \frac{m_l}{m_2} \Rightarrow \frac{p_l}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

3. **(b)** The centripetal acceleration

$a_c = k^2 r t^2 \Rightarrow \frac{v^2}{r} = k^2 r t^2$

$\Rightarrow \frac{1}{2}mv^2 = \frac{m}{2}k^2r^2t^2$ ...(i)

$\Rightarrow$ K.E. $= \frac{m}{2}k^2r^2t^2 \Rightarrow \frac{d}{dt}(\text{K.E.}) = mk^2r^2t$

$\Rightarrow$ Power $= mk^2r^2t$

4. **(b)** **KEY CONCEPT**

The force constant of a spring is inversely proportional to the length of the spring.

Let the original length of spring be L and spring constant is K (given)

Therefore,

$K \times L = \frac{2L}{3} \times K' \Rightarrow K' = \frac{3}{2}K$

5. **(c)** $F = v\left(\frac{dm}{dt}\right) = v\frac{d}{dt}(\rho \times \text{Volume}) = v\rho\frac{d}{dt}(\text{Volume})$

$= v\rho \times (Av) = A\rho v^2$

Power = Force × Velocity $= A\rho v^2 \times v = A\rho v^3$

$\Rightarrow P \propto v^3$

6. (d) $dU_{(x)} = -Fdx$

$$\therefore \quad U_x = -\int_0^x Fdx = \frac{kx^2}{2} - \frac{ax^4}{4}$$

$U = 0$ at $x = 0$ and at $x = \sqrt{\frac{2k}{a}}$; ⇒ we have potential energy zero twice (out of which one is at origin).

Also, when we put $x = 0$ in the given function, we get $F = 0$. But $F = -\frac{dU}{dx}$

⇒ At $x = 0$; $\frac{dU}{dx} = 0$ i.e. the slope of the graph should be zero. These characteristics are represented by (d).

7. (b) The above situation can also be looked upon as the decrease in the gravitational potential energy of spring mass system is equal to the gain in spring elastic potential energy.

$$Mgx = \frac{1}{2}kr^2, \quad x = \frac{2Mg}{k}$$

8. (b) **Note :** In a conservative field work done does not depend on the path. The gravitational field is a conservative field.

$\therefore \quad W_1 = W_2 = W_3$

9. (b) We know that $\Delta U = -W$ for conservative forces

$$\Delta U = -\int_0^x Fdx \text{ or } \Delta U = -\int_0^x kxdx$$

$$\Rightarrow \quad U_{(x)} - U_{(0)} = -\frac{kx^2}{2}$$

Given $\quad U_{(0)} = 0 \quad U_{(x)} = -\frac{kx^2}{2}$

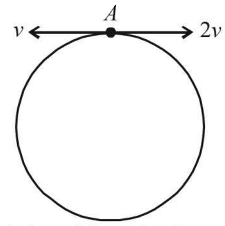
10. (c) When the block B is displaced towards wall 1, only spring S_1 is compressed and S_2 is in its natural state. This happens because the other end of S_2 is not attached to the wall but is free. Therefore the energy stored in the system $= \frac{1}{2}k_1 x^2$. When the block is released, it will come back to the equilibrium position, gain momentum, overshoot to equilibrium position and move towards wall 2. As this happens, the spring S_1 comes to its natural length and S_2 gets compressed. As there are no frictional forces involved, the P.E. stored in the spring S_1 gets stored as the P.E. of spring S_2 when the block B reaches its extreme position after compressing S_2 by y.

$$\therefore \frac{1}{2}k_1 x^2 = \frac{1}{2}k_2 y^2$$

$$\frac{1}{2} \times kx^2 = \frac{1}{2} 4ky^2, x^2 = 4y^2 \qquad \therefore \frac{y}{x} = \frac{1}{2}$$

11. (c) Let the radius of the circle be r. Then the two distance travelled by the two particles before first collision is $2\pi r$. Therefore

$$2v \times t + v \times t = 2\pi r$$

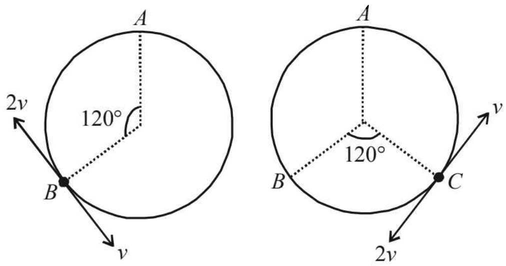


where t is the time taken for first collision to occur.

$$\therefore \quad t = \frac{2\pi r}{3v}$$

$\therefore$ Distance travelled by particle with velocity v is equal to $v \times \frac{2\pi r}{3v} = \frac{2\pi r}{3}$.

Therefore the collision occurs at B.



As the collision is elastic and the particles have equal masses, the velocities will interchange as shown in the figure. According to the same reasoning as above, the 2nd collision will take place at C and the velocities will again interchange.

With the same reasoning the 3rd collision will occur at the point A. Thus there will be two elastic collisions before the particles again reach at A.

12. (b) The forces acting on the bead as seen by the observer in the accelerated frame are : (a) N ; (b) mg ; (c) ma (pseudo force).

Let θ is the angle which the tangent at P makes with the X- axis. As the bead is in equilibrium with respect to the wire, therefore

$N \sin\theta = ma$ and $N \cos\theta = mg$

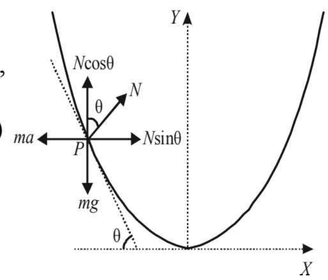
$\therefore \quad \tan\theta = \frac{a}{g} \ldots$ (i)

But $y = kx^2$. Therefore,

$\frac{dy}{dx} = 2kx = \tan\theta \ldots$ (ii)

From (i) & (ii)

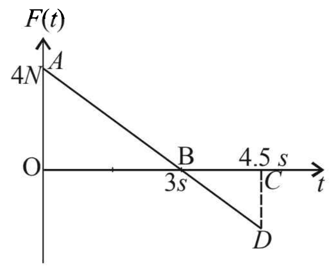
$$2kx = \frac{a}{g} \Rightarrow x = \frac{a}{2kg}$$



13. (c) Area under $F - t$ graph gives the impulse or the change in the linear momentum of the body. As the initial velocity (and therefore the initial linear momentum) of the body is zero, the area under $F - t$ graph gives the final linear momentum of the body.

Area of Δ AOB

$$= \frac{1}{2} \times 3 \times 4 = 6 \text{ N-s}$$



Also $\frac{OA}{OB} = \frac{CD}{CB}$

$$\Rightarrow \frac{4}{3} = \frac{CD}{1.5} f$$

$\Rightarrow CD = 2$

$\therefore$ Area of $\Delta BCD = = -\left[\frac{1}{2}\times 1.5\times 2\right] = -1.5$ N-s

$\therefore$ The final linear momentum $= 6 - 1.5 = 4.5$ N-s

$\therefore$ Kinetic energy of the block $= \frac{p^2}{2m} = \frac{(4.5)^2}{2\times 2} = 5.06$ J

14. **(d)** Let us consider a point on the circle

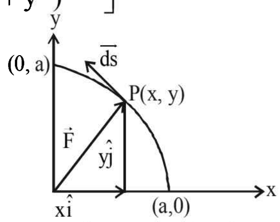
The equation of circle is $x^2 + y^2 = a^2$

The force is

$$\vec{F} = K\left[\frac{x\hat{i}}{(x^2+y^2)^{3/2}} + \frac{y\hat{j}}{(x^2+y^2)^{3/2}}\right]$$

$$\vec{F} = K\left[\frac{x\hat{i}}{(a^2)^{3/2}} + \frac{y\hat{j}}{(a^2)^{3/2}}\right]$$

$$\vec{F} = \frac{K}{a^3}\left[x\hat{i} + y\hat{j}\right]$$



The force acts radially outwards as shown in the figure and the displacement is tangential to the circular path. Therefore the angle between the force and displacement is 90° and W = 0

option (d) is correct.

15. **(b)** $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m[u+at]^2 = \frac{1}{2}m[0+gt]^2$

$\therefore K.E = \frac{1}{2}mgt^2 \qquad \therefore K.E \propto t^2 \qquad$ ...(1)

First the kinetic energy will increase as per eq (1). As the balls touches the ground it starts deforming and loses its K.E. (K.E. converting into elastic potential energy). When the deformation is maximum, K.E. = 0. The ball then again regain its shape when its elastic potential energy changes into K.E. As the ball moves up it loses K.E. and gain gravitational potential energy. These characteristics are according to graph (b).

D. MCQs with ONE or MORE THAN ONE Correct

1. **(c)** $P = \frac{E}{t} = \text{constt} \qquad \therefore \frac{\frac{1}{2}mv^2}{t} = \text{constt}$

$\Rightarrow \frac{v^2}{t} = \text{constt}\,(k) \therefore v = kt^{1/2}$ and $\frac{ds}{dt} = kt^{1/2}$

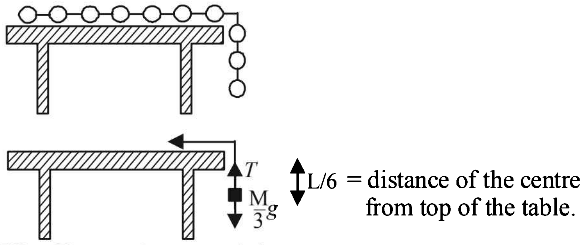
or, $ds = kt^{1/2}\,dt$

By integrating, we get

$\Rightarrow s = \frac{2kt^{3/2}}{3} + C \Rightarrow s \propto t^{3/2}$

2. **(d)** The hanging part of the chain which is to be pulled up can be considered as a point mass situated at the centre of the hanging part. The equivalent diagram is drawn.

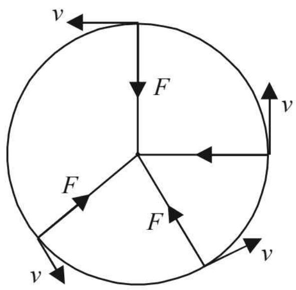
Note : The work done in bringing the mass up will be equal to the change in potential energy of the mass.



W = Change in potential energy

$$= mgh = \frac{M}{3}\times g\times\frac{L}{6} = \frac{MgL}{18}$$

3. **(c, d)** When the force is perpendicular to the velocity and constant in magnitude then the force acts as a centripetal force, and the body moves in a circular path. The force is constant in magnitude, this show the speed is not changing and hence kinetic energy will remain constant.



Note : The velocity changes continuously due to change in the direction. The acceleration also changes continuously due to change in direction.

4. **(c)** The expression of work done by the variable force F on the particle is given by

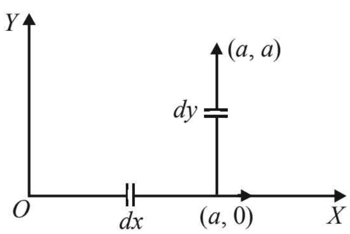
$$W = \int \vec{F}.\vec{d\ell}$$

In going from (0, 0) to $(a, 0)$, the coordinate of x varies from 0 to 'a', while that of y remains zero. Hence, the work done along this path is :

$$W_1 = \int_0^a (-Kx\hat{j}).dx\hat{i} = 0 \qquad [\because \hat{j}.\hat{i} = 0]$$

In going from $(a, 0)$ to (a, a) the coordinate of x remains constant ($= a$) while that of y changes from 0 to 'a'. Hence, the work done along this path is

$$W_2 = \int_0^a [(-K(y\hat{i} + a\hat{j}).dy\,\hat{j}] = ka\int_0^a dy = -Ka^2$$

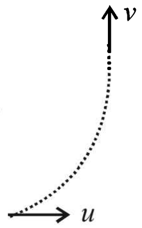


Hence, $W = W_1 + W_2 = -ka^2$

5. **(d)** Applying the principle of conservation of energy $(K.E.)_B + (P.E.)_B = (K.E.)_A + (P.E.)_A$, we get

$$\frac{1}{2}mv^2 + mgL = \frac{1}{2}mu^2$$

Hence, $v = \sqrt{u^2 - 2gL}$ ... (i)



Change in velocity $= |\vec{v} - \vec{u}| = \sqrt{v^2 + u^2}$

$= \sqrt{2(u^2 - g\ell)}$ [From (i)]

6. **(a, b)** At point A, potential energy of the ball = mgh_A
At point B, potential energy of the ball = 0
At point C, potential energy of the ball = mgh_C
Total energy at point A, $E_A = K_A + mgh_A$
Total energy at point B, $E_B = K_B$
Total energy at point C, $E_C = K_C + mgh_C$
According to the law of conservation of energy.
$E_A = E_B = E_C$ ... (i)
$E_A = E_B \Rightarrow E_C > K_C$ ...(ii)
$E_A = E_C$
$K_A + mgh_A = K_B + mgh_C$

or, $h_A - h_C = \dfrac{K_C - K_A}{mg}$ ...(iii)

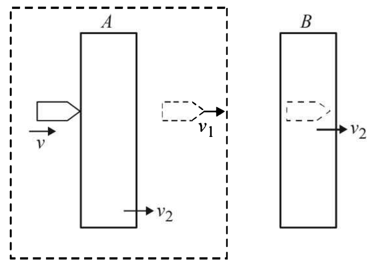
$\Rightarrow h_A > h_C; K_C > K_A$ ...(iv)
Option (b) is correct
From (i), (ii) and (iv), we get $h_A > h_C$; $K_B > K_C$. Option (a) is correct.

E. Subjective Problems

1. K.E. $= \dfrac{p^2}{2m}$ For equal value of p, K.E. $\propto \dfrac{1}{\text{mass}}$

2. **KEY CONCEPT**
For a spring, (spring constant) × (length) = Constant
If length is made one third, the spring constant becomes three times.

3. Let v be the velocity of bullet before striking A. Applying conservation of linear momentum for the system of bullet and plate A, we get
$0.02v = 0.02\,v_1 + 1 \times v_2$



Again applying conservation of linear momentum for collision at B.
$0.02\,v_1 = (2.98 + 0.02)\,v_2 = 3v_2$

$\Rightarrow v_2 = \dfrac{0.02\,v_1}{3}$ ... (ii)

From (i) and (ii)

$0.02\,v = 0.02\,v_1 + \dfrac{0.02\,v_1}{3}, v = \dfrac{4}{3}v_1 \Rightarrow \dfrac{v}{v_1} = \dfrac{4}{3}$

$\dfrac{v_1}{v} = \dfrac{3}{4} \Rightarrow 1 - \dfrac{v_1}{v} = 1 - \dfrac{3}{4} = \dfrac{1}{4} = 0.25 \Rightarrow \dfrac{v - v_1}{v} = 0.25$

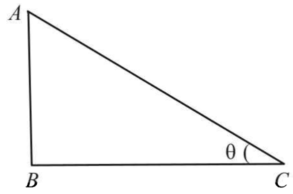
$\therefore$ % loss in velocity $= \dfrac{1}{4} \times 100 = 25\%$

4. No. An external force, the gravitational pull of earth, is acting on the ball which is responsible for the change in momentum.

5. **(a)** K.E. at C = Loss in P.E. – Work done by friction.

$\dfrac{1}{2}mv_c^2 = mg\,y - \mu mg\cos\theta \times AC$

$\therefore \dfrac{1}{2}v_c^2 = g\,y - \mu g\dfrac{BC}{AC} \times AC$



$= gy - \mu gx$

$\therefore v_c = \sqrt{2g(y - \mu x)}$

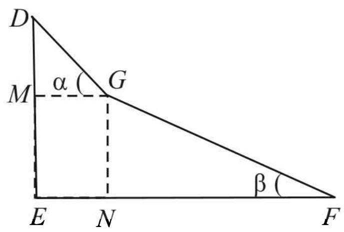
K.E. at F = loss in P.E. – Work done by friction

$\dfrac{1}{2}mv_F^2 = mgy - \mu mg\cos\alpha\,DG - \mu mg\cos\beta\,GF$

$\dfrac{1}{2}v_F^2 = gy - \mu g\dfrac{GM}{DG} \times DG - \mu g\dfrac{FN}{GF} \times GF$

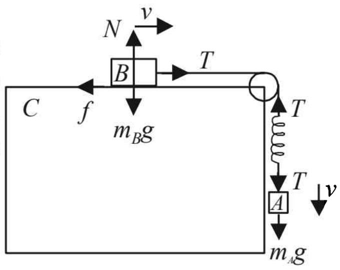
$\therefore \dfrac{1}{2}v_F^2 = gy - \mu g(GM + FN)$

$\therefore v_F = \sqrt{2g(y - \mu x)}$



Note : The result does not depend on the angles α and β. It only depends on the values of x and y.

6. Since the two blocks A and B are moving with constant velocity, therefore, the net force acting on A is zero and the net force acting on B will be zero. Since the spring is loaded, it will be in a deformed state. Let the extension of the spring be x.
The forces are drawn.



Note : There will be no friction force between block A and C
$\because$ $f = \mu N$. Here there is no normal reaction on A (because A is not pushing C)
Applying $F_{net} = ma$ on A, we get
$m_Ag - T = m_A \times 0$
$\therefore T = m_Ag$ ... (i)
Applying $F_{net} = ma$ on B, we get
$T - f = m_B \times 0$
$\therefore T = f = \mu N$
$= \mu m_Bg$ ... (ii)

From (i) and (ii)

$$\mu m_B g = m_A g \Rightarrow m_B = \frac{m_A}{\mu} = \frac{2}{0.2} = 10\,\text{kg}$$

Here the force acting on the spring is the tension (equal to restoring force)

$$\therefore \quad T = kx \qquad \therefore \quad x = \frac{T}{k}$$

$$\therefore \quad x = \frac{19.6}{k} \qquad [\because T = 2 \times 9.8 = 19.6\,N \text{ from (i)}]$$

The P.E. stored in spring is given by

$$U = \frac{1}{2}kx^2 = \frac{1}{2} \times k \times \frac{19.6}{k} \times \frac{19.6}{k}$$

$$= \frac{19.6 \times 19.6}{2 \times 1960} = 0.098\,\text{J}$$

7. K.E. of block = work against friction + P.E. of spring

$$\frac{1}{2}mv^2 = \mu_k mg(2.14 + x) + \frac{1}{2}kx^2$$

$$\frac{1}{2} \times 0.5 \times 3^2 = 0.2 \times 0.5 \times 9.8(2.14 + x) + \frac{1}{2} 2 \times x^2$$

$$2.14 + x + x^2 = 2.25$$

$$\therefore \quad x^2 + x - 0.11 = 0$$

On solving, we get $x = -\frac{11}{10}$ or $x = \frac{1}{10} = 0.1$ (valid answer)

Here the body stops momentarily.

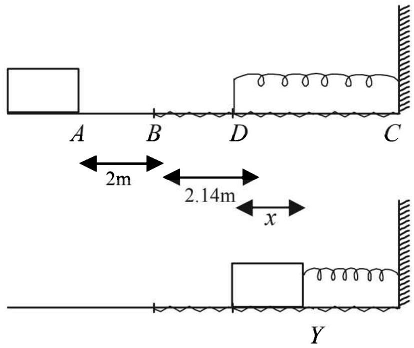
Restoring force at $Y = kx = 2 \times 0.1 = 0.2\,N$

Frictional force at $Y = \mu_s mg = 0.22 \times 0.5 \times 9.8 = 1.078\,N$

Since frictional force > restoring force, the body will stop here.

$\therefore$ The total distance travelled

$= AB + BD + DY = 2 + 2.14 + 0.1 = 4.24$ m.



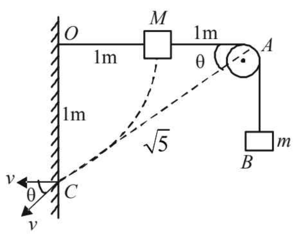
8. When mass m is released, since $M > m$, the mass M will move on a dotted path with O as the centre. There will be decrease in the potential energy of M which will be converted into kinetic energy of M, and increase in potential energy of m.

Decrease in P.E. of M is Mgh

$= 2 \times 9.8 \times 1 = 19.6$ J

K.E. of $M = \frac{1}{2}MV^2$

(Let V be the velocity attained by M just before striking the wall)

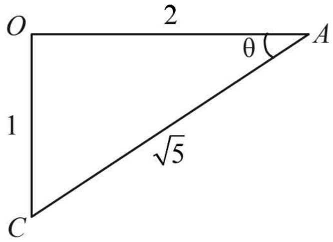


K.E. of $m = \frac{1}{2}mv^2$

From the figure, by velocity constraint

$v = V\cos\theta$

From ΔOAC,



$$\cos\theta = \frac{2}{\sqrt{5}}$$

$$\therefore \quad v = \frac{2V}{\sqrt{5}}$$

$(OC + CA) - OA$ = height attained by m

$1 + \sqrt{2^2 + 1^2} - 2 =$ height attained by $m = \sqrt{5} - 1$

$\therefore$ Increase in P.E. of $m = mgh' = 0.5 \times 9.8(\sqrt{5} - 1)$

NOTE THIS STEP

By the principle of energy conservation

$$Mgh = \frac{1}{2}MV^2 + \frac{1}{2}mv^2 + mgh'$$

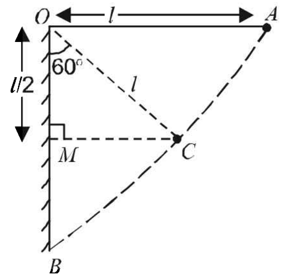
$$= \frac{1}{2}MV^2 + \frac{1}{2}m(V\cos\theta)^2 + mgh'$$

$$\therefore \quad 19.6 = \frac{1}{2} \times 2 \times V^2 + \frac{1}{2} 0.5 \times \frac{4V^2}{5} + 0.5 \times 9.8(\sqrt{5} - 1)$$

On solving, we get $V = 3.29$ m/s

9. The pendulum bob is left from position A. When it is at position C, the angular amplitude is 60°.

In ΔOCM



$$\cos 60° = \frac{OM}{\ell} \Rightarrow OM = \frac{\ell}{2}$$

The velocity of bob at B,

v_B before first collision is

$$mg\ell = \frac{1}{2}mv_B^2 \Rightarrow v_B = \sqrt{2g\ell}$$

Let after n collisions, the angular amplitude is 60° when the bob again moves towards the wall from C, the velocity v'_B before collision is

$$mg\frac{\ell}{2} = \frac{1}{2}mv'^2_B \Rightarrow v'_B = \sqrt{g\ell}$$

This means that the velocity of the bob should reduce from $\sqrt{2g\ell}$ to $\sqrt{g\ell}$ due to collisions with walls.

The final velocity after n collisions is $\sqrt{g\ell}$

$$\therefore \quad e^n(\sqrt{2g\ell}) = \sqrt{g\ell}$$

where e is coefficient of restitution.

$$\left(\frac{2}{\sqrt{5}}\right)^n \times \sqrt{2g\ell} = \sqrt{g\ell} \Rightarrow \left(\frac{2}{\sqrt{5}}\right)^n = \frac{1}{\sqrt{2}}$$

Taking log on both sides we get

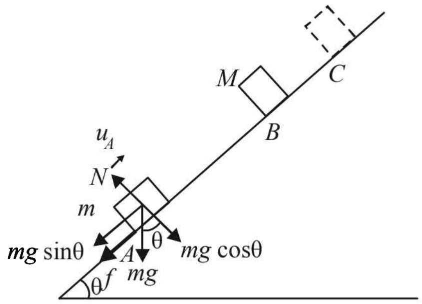
$$n \log\left(\frac{2}{\sqrt{5}}\right) = \log\frac{1}{\sqrt{2}} \Rightarrow n = 3.1$$

Therefore, number of collisions will be 4.

10. From **A** to **B**.

$u = 10$ m/s (given)

$$a = -\left[\frac{mg\sin\theta + f}{m}\right] = -\left[\frac{mg\sin\theta + \mu mg\cos\theta}{m}\right]$$



$= -[g \sin\theta + \mu g \cos\theta] = -g[\sin\theta + \mu\cos\theta]$

$= -10[0.05 + 0.25 \times 0.99] = -2.99 \text{ m/s}^2$

$v = ?$

$s = 6 \text{ m}$

$v^2 - u^2 = 2as \Rightarrow v^2 = 100 + 2(-2.99) \times 6 \Rightarrow v = 8 \text{ m/s}$

$\Rightarrow$ The velocity of mass m just before collision is 8 m/s.

The velocity of mass M just before collision is 0 m/s (given).

AFTER COLLISION

Let v_1 be the velocity of mass m after collision and v_2 be the velocity of mass M after collision. **Body of mass M moving from B to C and coming to rest.**

$u = v_2$

$v = 0$

$a = -2.99 \text{ m/s}^2$

(same as of previous case because all other things are same except mass. a is independent of mass) $s = 0.5$

$v^2 - u^2 = 2as \Rightarrow (0)^2 - v_2^2 = 2(-2.99) \times 0.5$

$\Rightarrow v_2 = 1.73 \text{ m/s}$

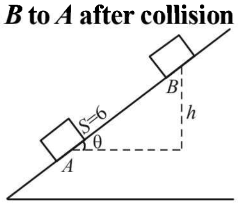
Body of mass m moving from B to A after collision

$\sin\theta = \frac{h}{6}$

$h = 6\sin\theta = 6 \times 0.05$

$u = v_1$

$v = +1 \text{ m/s}$



$(\text{K.E.} + \text{P.E.})_{\text{initial}} = (\text{K.E.} + \text{P.E.})_{\text{final}} + W_{\text{friction}}$

$$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv^2 + 0 + \mu mgs$$

$$\frac{1}{2}v_1^2 + 10 \times (6 \times 0.05) = \frac{1}{2}(1)^2 + 0.25 \times 10 \times 6$$

$v_1 = -5 \text{ m/s}$

$\therefore$ **Coefficient of restitution**

$$e = \left|\frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}\right|$$

$$= \left|\frac{-5 - 1.73}{8 - 0}\right| = 0.84$$

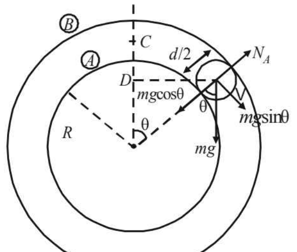
On applying conservation of linear momentum before and after collision, we get

$2 \times 8 + M \times 0 = 2 \times (-5) + M(1.73)$

$$\therefore \quad M = \frac{26}{1.73} = 15.02 \text{ kg}$$

11. The ball is moving in a circular motion. The necessary centripetal force is provided by $(mg\cos\theta - N)$. Therefore,

$$mg\sin\theta - N_A = \frac{mv^2}{\left(R + \frac{d}{2}\right)} \quad \text{... (i)}$$



According to energy conservation

$$\frac{1}{2}mv^2 = mg\left(R + \frac{d}{2}\right)(1 - \cos\theta) \quad \text{... (ii)}$$

From (i) and (ii)

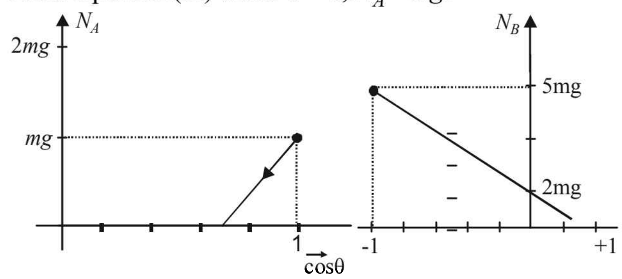
$N_A = mg(3\cos\theta - 2) \quad \text{... (iii)}$

The above equation shows that as θ increases N_A decreases. At a particular value of θ, N_A will become zero and the ball will lose contact with sphere A. This condition can be found by putting $N_A = 0$ in eq. (iii)

$$0 = mg(3\cos\theta - 2) \therefore \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

The graph between N_A and $\cos\theta$

From equation (iii) when $\theta = 0$, $N_A = mg$.



When $\theta = \cos^{-1}\left(\frac{2}{3}\right)$; $N_A = 0$

The graph is a straight line as shown.

when $\theta > \cos^{-1}\left(\frac{2}{3}\right)$; $N_B - (mg\cos\theta) = \frac{mv^2}{R + \frac{d}{2}}$

$$\Rightarrow N_B + mg\cos\theta = \frac{mv^2}{\left(R + \frac{d}{2}\right)} \quad \text{... (iv)}$$

Using energy conservation

$$\frac{1}{2}mv^2 = mg\left[\left(R + \frac{d}{2}\right) - \left(R + \frac{d}{2}\right)\cos\theta\right]$$

$$\frac{mv^2}{\left(R + \frac{d}{2}\right)} = 2mg[1 - \cos\theta] \quad \text{... (v)}$$

From (iv) and (v), we get

$N_B + mg\cos\theta = 2mg - 2mg\cos\theta$

$N_B = mg(2 - 3\cos\theta)$

When $\cos\theta = \frac{2}{3}$, $N_B = 0$

When $\cos\theta = -1$, $N_B = 5\,mg$

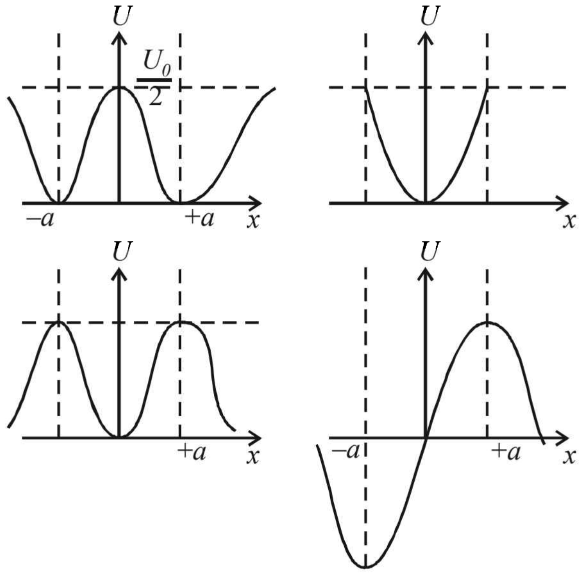
Therefore the graph is as shown.

F. Match the Following

1. A → p, q, r, t; B → q, s; C → p, q, r, s; D → p, r, t

For A

$$Fx = -\frac{dU}{dx} = \frac{-d}{dx}\left[\frac{U_0}{2}\left(1-\left(\frac{x}{a}\right)^2\right)^2\right] = \frac{-2U_0}{a^3}(x-a)x(x+a)$$



For B $F_x = \frac{-dU}{dx} = -U_0\left(\frac{x}{a}\right)$

For C $F_x = \frac{-dU}{dx} = U_0\frac{e^{-x^2/a^2}}{a^3}x(x-a)(x+a)$

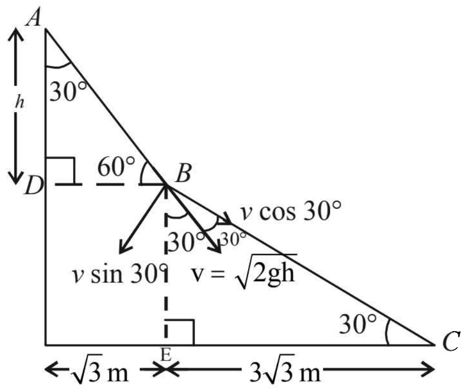
For D $F_x = -\frac{dU}{dx} = -\frac{U_0}{2a^3}[(x-a)(x+a)]$

G. Comprehension Based Questions

1. (b) As the inclined plane is frictionless,

The K. E . at B = P.E. at *A*

$\frac{1}{2}mv^2 = mgh \Rightarrow v = \sqrt{2gh}$



In Δ*ADB*, $\tan 60° = \frac{h}{\sqrt{3}}$

$\therefore h = 3$ m

$\therefore v = \sqrt{6g} = \sqrt{60}$ m/s

This is the velocity of the block just before collision. This velocity makes an angle of 30° with the vertical.

Also in right angled triangle *BEC*, ∠*EBC*= 60°. Therefore v makes an angle of 30° with the second inclined plane *BC*. The component of v along *BC* is *v* cos 30°.

It is given that the collision at *B* is perfectly inelastic therefore the impact forces act normal to the plane such that the vertical component of velocity becomes zero. The component of velocity along the incline *BC* remains unchanged and is equal to *v* cos 30°

$= \sqrt{60} \times \frac{\sqrt{3}}{2} = \sqrt{\frac{180}{4}} = \sqrt{45}$ m/s

2. (b) In Δ*BCE*, $\tan 30° = \frac{BE}{CE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BE}{3\sqrt{3}} \Rightarrow BE = 3$m

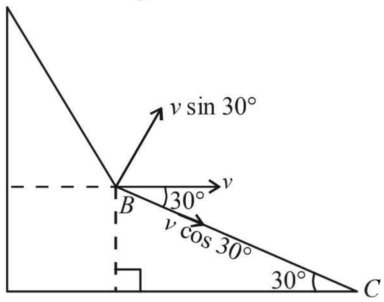
Applying mechanical energy conservation .

Mechanical energy at *B* = Mechanical energy at *C*

$\frac{1}{2}M\left(\sqrt{45}\right)^2 + M \times 10 \times 3 = \frac{1}{2}Mv_c^2$

$45 + 60 = v_c^2 \quad \therefore v_c = \sqrt{105}$ m/s

3. (c) The velocity of the block along *BC* just before collision is v cos 30°. The impact forces act perpendicular to the surface so the component of velocity along the incline remains unchanged.

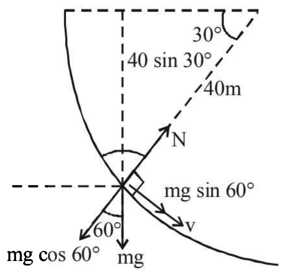


Just after collision

Also since the collision is elastic, the vertical component of velocity (*v* sin 30°) before collision changes in direction, the magnitude remaining the same as shown in the figure. So the rectangular components of velocity after collision are as shown in the figure. This means that the final velocity of the block should be horizontal making an angle 30° with *BC*. Therefore the vertical component of the final velocity of the block is zero.

4. (a) $N - mg\cos 60° = \frac{mv^2}{r}$

$\therefore N = mg\cos 60° + \frac{mv^2}{r}$ ...(1)



Loss in P.E. = mg × 40 sin 30° = 200 J

Work done in over coming friction = 150 J

∴ K.E. possessed by the particle = 50 J

∴ $\frac{1}{2}mv^2 = 50$J

∴ $mv^2 = 100$ J ...(2)

From (1) and (2), $N = 1 \times 10 \times \frac{1}{2} + \frac{100}{40} = 5 + 2.5 = 7.5\,N$

(a) is the correct option.

5. (b) From (2), $mv^2 = 100 \quad \therefore v = 10\ ms^{-1}$

(b) is the correct option.

H. Assertion & Reason Type Questions

1. (c) **Statement 1 :** In the first case the mechanical energy is completely converted into heat becuase of fiction. While is second case, a part of mechanical energy is converted into heat due to fiction but another part of mechanical energy is retained in the form of potential energy of the block.

Therefore statement 1 is correct.

Statement 2 : This is a wrong statement because the coefficient of friction between the block and the surface does not depend on the angle of inclination.

I. Integer Value Correct Type

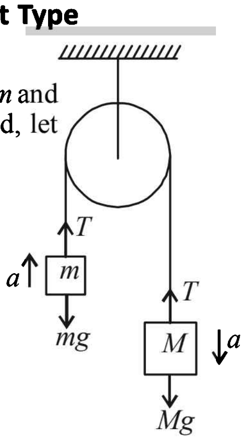
1. 8 Given $m = 0.36$ kg, $M = 0.72$ kg.

The figure shows the forces on m and M. When the system is released, let the acceleration be a. Then

$T - mg = ma$

$Mg - T = Ma$

$\therefore \quad a = \frac{(M-m)g}{M+m} = g/3$



and $T = 4\,mg/3$

For block m :

$u = 0, a = g/3, t = 1, s = ?$

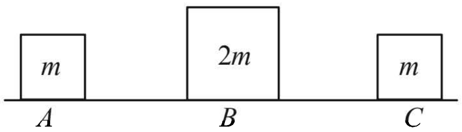
$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times \frac{g}{3} \times 1^2 = g/6$

$\therefore$ Work done by the string on m is

$\vec{T}.\vec{s} = Ts = 4\frac{mg}{3} \times \frac{g}{6} = \frac{4 \times 0.36 \times 10 \times 10}{3 \times 6} = 8\,J$

2. 4 The velocity of B just after collision with A is

$$v_B = \frac{(m_B - m_A)u_B}{m_B + m_A} + \frac{2m_A u_A}{m_A + m_B}$$



$$= \frac{0 + 2m \times 9}{m + 2m} = 6\text{m/s}$$

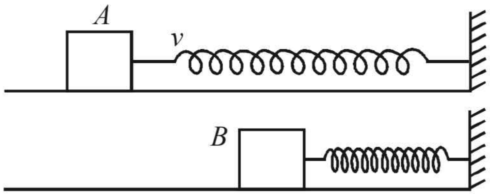
The collision between B and C is completely inelastic.

$\therefore \quad m_B v_B = (m_B + m_c)v$

$\therefore \quad v = \frac{6 \times 2m}{2m + m} = 4\text{m/s}.$

3. 4

Let v be the speed of the block just after impulse. At B, the block comes to rest. Therefore



Loss in K.E. of the block = Gain in P.E. of the spring + Work done against friction

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \mu mg.x$$

$$\therefore v = \sqrt{\frac{k}{m}x^2 + \mu gx}.$$

$$v = \sqrt{\frac{2}{0.18} \times 0.06 \times 0.06 + 0.1 \times 10 \times 0.06}$$

$\therefore v = \frac{4}{10} \qquad \therefore N = 4$

4. 5 Here Δ K.E. = W = P × t

$$\therefore \frac{1}{2}mv^2 = P \times t$$

$$\therefore v = \sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2 \times 0.5 \times 5}{0.2}} = 5ms^{-1}$$

5. 5 Work done = Increase in potential energy + gain in kinetic energy

$F \times d = mgh$ + gain in K.E.

$18 \times 5 = 1 \times 10 \times 4$ + gain in K.E.

$\therefore$ Gain in K.E. = 50 J = 10n

$\therefore n = 5$

Section-B JEE Main/ AIEEE

1. (c) Kinetic energy of a system of particle is zero only when the speed of each particles is zero. And if speed of each particle is zero, the linear momentum of the system of particle has to be zero.

Also the linear momentum of the system may be zero even when the particles are moving. This is because linear momentum is a vector quantity. In this case the kinetic energy of the system of particles will not be zero.

$\therefore$ A does not imply B but B implies A.

2. (d) The elastic potential energy

$$= \frac{1}{2} \times \text{Force} \times \text{extension} = \frac{1}{2} \times 200 \times 0.001 = 0.1\text{ J}$$

3. (b) $k = 5 \times 10^3$ N/m

$$W = \frac{1}{2}k\left(x_2^2 - x_1^2\right) = \frac{1}{2} \times 5 \times 10^3\left[(0.1)^2 - (0.05)^2\right]$$

$= \frac{5000}{2} \times 0.15 \times 0.05 = 18.75$ Nm

4. **(b)** We know that $F \times v =$ Power

$\therefore F \times v = c$ where c = constant

$\therefore m\frac{dv}{dt} \times v = c \qquad \left(\because F = ma = \frac{mdv}{dt}\right)$

$\therefore m\int_0^v vdv = c\int_0^t dt \qquad \therefore \frac{1}{2}mv^2 = ct$

$\therefore v = \sqrt{\frac{2c}{m}} \times t^{1/2}$

$\therefore \frac{dx}{dt} = \sqrt{\frac{2c}{m}} \times t^{1/2}$ where $v = \frac{dx}{dt}$

$\therefore \int_0^x dx = \sqrt{\frac{2c}{m}} \times \int_0^t t^{1/2}dt$

$x = \sqrt{\frac{2c}{m}} \times \frac{2t^{3/2}}{3} \Rightarrow x \propto t^{3/2}$

5. **(c)** Given : retardation $\propto$ displacement

i.e., $a = -kx$

But $a = v\frac{dv}{dx} \qquad \therefore \frac{vdv}{dx} = -kx \Rightarrow \int_{v_1}^{v_2} v\,dv = -k\int_0^x xdx$

$\left(v_2^2 - v_1^2\right) = -k\frac{x^2}{2}$

$\Rightarrow \frac{1}{2}m\left(v_2^2 - v_1^2\right) = \frac{1}{2}mk\left(\frac{-x^2}{2}\right)$

$\therefore$ Loss in kinetic energy, $\therefore \Delta K \propto x^2$

6. **(b)** Mass of over hanging chain $m' = \frac{4}{2} \times (0.6)$kg

Let at the surface PE = 0

C.M.of hanging part = 0.3 m below the table

$U_i = -m'gx = -\frac{4}{2} \times 0.6 \times 10 \times 0.30$

$\Delta U = m'gx = 3.6$J = Workdone in putting the entire chain on the table.

7. **(b)** Workdone in displacing the particle,

$W = \vec{F}.\vec{x} = (5\hat{i} + 3\hat{j} + 2\hat{k}).(2\hat{i} - \hat{j})$

$= 10 - 3 = 7$ joules

8. **(b)** Let acceleration of body be a

$\therefore v_1 = 0 + at_1 \Rightarrow a = \frac{v_1}{t_1}$

$\therefore v = at \Rightarrow v = \frac{v_1 t}{t_1}$

$P_{inst} = \vec{F}.\vec{v} = (m\vec{a}).\vec{v}$

$= \left(\frac{mv_1}{t_1}\right)\left(\frac{v_1 t}{t_1}\right) = m\left(\frac{v_1}{t_1}\right)^2 t$

9. **(a)** Work done by such force is always zero since force is acting in a direction perpendicular to velocity.

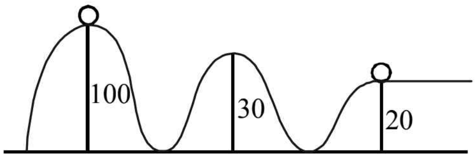
$\therefore$ from work-energy theorem = $\Delta K = 0$

K remains constant.

10. **(b)** $\frac{1}{2}Mv^2 = \frac{1}{2}kL^2 \Rightarrow v = \sqrt{\frac{k}{M}}.L$

Momentum $= M \times v = M \times \sqrt{\frac{k}{M}}.L = \sqrt{kM}.L$

11. **(b)**



Loss in potential energy = gain in kinetic energy

$m \times g \times 80 = \frac{1}{2}mv^2, \; 10 \times 80 = \frac{1}{2}v^2$

$v^2 = 1600$ or $v = 40$ m/s

12. **(b)** $u = 0; v = u + aT; v = aT$

Instantaneous power $= F \times v = m.a.at = m.a^2.t$

$\therefore$ Instantaneous power $= m\frac{v^2}{T^2}t$

13. **(b)** $K.E = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.1 \times 25 = 1.25$ J

$W = -mgh = -\left(\frac{1}{2}mv^2\right) = -1.25$ J

$\left[\because mgh = \frac{1}{2}mv^2 \text{ by energy conservation}\right]$

14. **(a)** Velocity is maximum when K.E. is maximum

For minimum. P.E.,

$\frac{dV}{dx} = 0 \Rightarrow x^3 - x = 0 \Rightarrow x = \pm 1$

$\Rightarrow$ Min. P.E. $= \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$ J

$K.E._{(max.)} + P.E._{(min.)} = 2$ (Given)

$\therefore K.E._{(max.)} = 2 + \frac{1}{4} = \frac{9}{4} = \frac{1}{2}mv^2_{max.}$

$\Rightarrow \frac{1}{2} \times 1 \times v^2_{max.} = \frac{9}{4} \Rightarrow v_{max.} = \frac{3}{\sqrt{2}}$

15. **(b)** Let the block compress the spring by x before stopping.

kinetic energy of the block = (P.E of compressed spring) + work done against friction.

$\frac{1}{2} \times 2 \times (4)^2 = \frac{1}{2} \times 10{,}000 \times x^2 + 15 \times x$

$10{,}000x^2 + 30x - 32 = 0$

$\Rightarrow 5000x^2 + 15x - 16 = 0$

$\therefore x = \frac{-15 \pm \sqrt{(15)^2 - 4 \times (5000)(-16)}}{2 \times 5000}$

$= 0.055$m $= 5.5$cm.

16. (d) The average speed of the athelete

$$v=\frac{100}{10}=10\,\text{m/s} \quad \therefore \text{ K.E.}=\frac{1}{2}mv^2$$

If mass is 40 kg then, $\text{K.E.}=\frac{1}{2}\times 40\times(10)^2=2000\,\text{J}$

If mass is 100 kg then, $\text{K.E.}=\frac{1}{2}\times 100\times(10)^2=5000\,\text{J}$

17. (c) Initial kinetic energy of the system

$$\text{K.E}_i=\frac{1}{2}mu^2+\frac{1}{2}M(0)^2=\frac{1}{2}\times 0.5\times 2\times 2+0=1\text{J}$$

For collision, applying conservation of linear momentum

$$m\times u=(m+M)\times v$$

$$\therefore\ 0.5\times 2=(0.5+1)\times v \Rightarrow v=\frac{2}{3}\text{m/s}$$

Final kinetic energy of the system is

$$\text{K.E}_f=\frac{1}{2}(m+M)v^2=\frac{1}{2}(0.5+1)\times\frac{2}{3}\times\frac{2}{3}=\frac{1}{3}\text{J}$$

$$\therefore \text{ Energy loss during collision }=\left(1-\frac{1}{3}\right)\text{J}=0.67\text{J}$$

18. (c) At equilibrium : $\frac{dU(x)}{dx}=0$

$$\Rightarrow \frac{-12a}{x^{11}}=\frac{-6b}{x^5} \Rightarrow x=\left(\frac{2a}{b}\right)^{\frac{1}{6}}$$

$$\therefore U_{\text{at equilibrium}}=\frac{a}{\left(\frac{2a}{b}\right)^2}-\frac{b}{\left(\frac{2a}{b}\right)}=-\frac{b^2}{4a} \text{ and } U_{(x=\infty)}=0$$

$$\therefore D=0-\left(-\frac{b^2}{4a}\right)=\frac{b^2}{4a}$$

19. (a) When force is same

$$W=\frac{1}{2}kx^2$$

$$W=\frac{1}{2}k\frac{F^2}{k^2} \quad [\because F=kx]$$

$$\therefore W=\frac{F^2}{2x}$$

As $W_1 > W_2$

$\therefore k_1 < k_2$

When extension is same

$W \propto k$ ($\because x$ is same)

$\therefore W_1 < W_2$

Statement 1 is false and statement 2 is true.

20. (c) Work done in stretching the rubber-band by a distance dx is

$$dW=F\,dx=(ax+bx^2)dx$$

Integrating both sides,

$$W=\int_0^L ax\,dx+\int_0^L bx^2\,dx=\frac{aL^2}{2}+\frac{bL^3}{3}$$

21. (b) $\text{n}=\frac{\text{W}}{\text{input}}=\frac{\text{mgh}\times 1000}{\text{input}}=\frac{10\times 9.8\times 1\times 1000}{\text{input}}$

$$\text{Input}=\frac{98000}{0.2}=49\times 10^4\text{J}$$

$$\text{Fat used}=\frac{49\times 10^4}{3.8\times 10^7}=12.89\times 10^{-3}\text{kg.}$$